A.1 Calculus (limit, Continuity)

1. Suppose $\lim_{x\to a} f(x) > 0$. Show that there exists $\delta > 0$ such that

$$0 < |x-a| < \delta \Rightarrow f(x) > 0$$

2. Suppose that $\lim_{x\to a} f(x) = l > 0$. Show that there exists $\delta > 0$ such that $0 < |x-a| < \delta \Rightarrow f(x) > \frac{l}{2}$.

3.
$$f(x) = \begin{cases} ax^2 + b & x < 1 \\ 2 & x = 1 \\ x + b & x > 1 \end{cases}$$

f is continuous at 1. Find a, b.

4. Use ε-δ definition to show "Sandwich Theorem" (i.e. Squeezing Theorem).

5. State Intermediate Value Theorem (for Continuous Functions).

6. Give definitions of the following.

(1) $\lim_{x\to\infty} f(x) = l$ (2) $\lim_{x\to\infty} f(x) = \infty$ (3) $\lim_{x\to\infty} f(x) = -\infty$

(4) $\lim_{x\to-\infty} f(x) = -\infty$